LIBERTY PAPER SET

STD. 10 : Mathematics (Standard) [N-012(E)]

Full Solution

Time : 3 Hours **ASSIGNTMENT PAPER 1**

Section-A

1. (B) 2 **2.** (A) 1 **3.** (C) Parallel lines **4.** (D) 4 **5.** (C) –77 **6.** (C) 4.5 **7.** –1 **8.** 0 **9.** Secant **10.** 30° **11.** $\pi rl + \pi r^2$ **12.** $\frac{1}{2}$ **13.** False **14.** False **15.** True **16.** False **17.** 21% **18.** 5.5 **19.** 9 **20.** 0 **21.** (a) $\frac{-b}{a}$ **22.** (c) $\frac{-b}{c}$ **23.** (c) $\sqrt{\frac{1}{4}}$ **24.** (a) 1

Section-B

25. Let's $a = 306$ and $b = 657$

$$\text{LCM}(a, b) = \frac{a \times b}{\text{HCF}(a, b)}$$

$$\therefore \text{LCM}(306, 657) = \frac{306 \times 657}{\text{HCF}(306, 657)}$$

$$= \frac{306 \times 657}{9}$$

$$= 22338$$

26. By the method of elimination :

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$\therefore 3x + 4y = -6 \quad \text{...(1)}$$

$$x - \frac{y}{3} = 3$$

$$\therefore 3x - y = 9 \quad \text{...(2)}$$

Subtract equation (1) and (2),

$$\begin{array}{rcl} 3x + 4y & = & -6 \\ 3x - y & = & 9 \\ - \quad + & & - \\ \hline \end{array}$$

$$\therefore 5y = -15$$

$$\therefore y = -3$$

Put $y = -3$ in equation (2)

$$3x - y = 9$$

$$\therefore 3x - (-3) = 9$$

$$\therefore 3x + 3 = 9$$

$$\therefore 3x = 6$$

$$\therefore x = 2$$

The solution of the equation : $x = 2$, $y = -3$

27. $\therefore \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\therefore \sqrt{2}x^2 + 5x + \sqrt{2}\sqrt{2}x + 5\sqrt{2} = 0$

$\therefore x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\therefore (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$\therefore x + \sqrt{2} = 0$ OR $\sqrt{2}x + 5 = 0$

$\therefore x = -\sqrt{2}$ OR $\sqrt{2}x = -5$

$$x = \frac{-5}{\sqrt{2}}$$

$\therefore$ The roots of this equation : $-\sqrt{2}, \frac{-5}{\sqrt{2}}$

28. $2x^2 - 6x + 3 = 0$

$\therefore a = 2, b = -6$ and $c = 3$

$\therefore b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12$

Here $b^2 - 4ac > 0$, therefore, there are distinct real roots exist for given equation.

Now, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-(-6) \pm \sqrt{12}}{2 \times 2}$

$\therefore x = \frac{6 \pm 2\sqrt{3}}{4}$

$\therefore x = \frac{3 \pm \sqrt{3}}{2}$

Therefore, roots of given equation : $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

29. Here, $a_3 = 5$

$\therefore a + 2d = 5$ ...(1)

$a_7 = 9$

$\therefore a + 6d = 9$ ..(2)

Subtract equation (2) by (1),

$(a + 2d) - (a + 6d) = 5 - 9$

$\therefore a + 2d - a - 6d = -4$

$\therefore -4d = -4$

$\therefore d = 1$

Put $d = 1$ in equation (1),

$a + 2d = 5$

$\therefore a + 2(1) = 5$

$\therefore a + 2 = 5$

$\therefore a = 3$

$\therefore a_1 = a = 3$

$a_2 = a + d = 3 + 1 = 4$

$a_3 = a + 2d = 3 + 2(1) = 3 + 2 = 5$

$a_4 = a + 3d = 3 + 3(1) = 3 + 3 = 6$

Hence, the required AP is 3, 4, 5, 6, 7, ...

30. $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 + \frac{3}{4} - \frac{3}{4}$

$= 2$

31. LHS $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A \cdot (1+\sin A)}$

$= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{\cos A \cdot (1+\sin A)}$

$= \frac{1+1+2\sin A}{\cos A \cdot (1+\sin A)}$

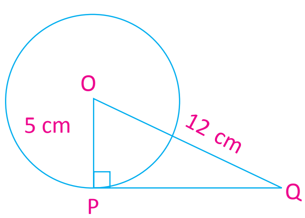
$= \frac{2+2\sin A}{\cos A \cdot (1+\sin A)}$

$= \frac{2(1+\sin A)}{\cos A \cdot (1+\sin A)}$

$= \frac{2}{\cos A}$

$= 2\sec A =$ RHS

32. In Δ OPQ; $\angle P = 90°$



Applying Pythagoras Theorem,

$OQ^2 = OP^2 + PQ^2$

$\therefore PQ^2 = OQ^2 - OP^2$

$\therefore PQ^2 = (12)^2 - (5)^2$

$\therefore PQ^2 = 144 - 25$

$\therefore PQ^2 = 119$

$\therefore PQ = \sqrt{119}$ m

33. Suppose, the side length of the cube be x.

$\therefore$ Volume of cube $= x^3$

$\therefore 64 = x^3$

$\therefore x = 4$ cm

$l = 2x = 2 \times 4 = 8$ cm, $b = x = 4$ cm and

$h = x = 4$ cm

$\therefore$ Area of cuboid $= 2\,(lb + bh + hl)$

$= 2\,(8 \times 4 + 4 \times 4 + 4 \times 8)$

$= 2\,(32 + 16 + 32)$

$= 2(80)$

$= 160\ cm^2$

34. Here the maximum class frequency is 8, and the class corresponding to this frequency is 3 – 5. So, the modal class is 3 – 5.

$\therefore$ l = Lower limit of modal class = 3

h = Class size = 2

f_1 = frequency of the modal class = 8

f_0 = frequency of class preceding the modal class = 7

f_2 = frequency of class succeeding the modal class = 2

Mode $\quad Z = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$\therefore Z = 3 + \left(\dfrac{8-7}{2(8)-7-2}\right) \times 2$

$\therefore Z = 3 + \dfrac{1}{7} \times 2 = 3 + \dfrac{2}{7}$

$\therefore Z = 3.286$

Therefore, the mode of the data above is 3.286.

35.

Monthly consumption (in units) (class)	f_i	x_i	u_i	$f_i u_i$
65 – 85	4	75	– 3	– 12
85 – 105	5	95	– 2	– 10
105 – 125	13	115	– 1	– 13
125 – 145	20	135 = a	0	0
145 – 165	14	155	1	14
165 – 185	8	175	2	16
185 – 205	4	195	3	12
Total	68	–	–	7

$a = 135$, $h = 20$

Mean $\bar{x} = a + \left(\dfrac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$

$\therefore \bar{x} = 135 + \dfrac{7}{68} \times 20$

$\therefore \bar{x} = 135 + 2.05$

$\therefore \bar{x} = 137.05$ unit

36. $P(A) + P(\bar{A}) = 1$

$\therefore (0.8)^2 + P(\bar{A}) = 1$

$\therefore 0.64 + P(\bar{A}) = 1$

$\therefore P(\bar{A}) = 1 - 0.64$

$\therefore P(\bar{A}) = 0.36$

37. Possible outcomes in throwing a dice are = 6

(1, 2, 3, 4, 5, 6)

(i) Suppose event A gets a prime number on the die.

$\therefore$ P(A) = $\dfrac{\text{Number of prime number}}{\text{Total number of possible outcomes}}$

$\therefore$ P(A) = $\dfrac{3}{6}$

$\therefore$ P(A) = $\dfrac{1}{2}$

(ii) Suppose event B getting a number of odd number on dice.

$\therefore$ P(B) = $\dfrac{\text{Number of odd number}}{\text{Total number of possible outcomes}}$

$\therefore$ P(B) = $\dfrac{3}{6}$

$\therefore$ P(B) = $\dfrac{1}{2}$

Section-C

38. $x^2 - 5 = 0$

$\therefore x^2 - (\sqrt{5})^2 = 0$

$\therefore (x - \sqrt{5})(x + \sqrt{5}) = 0$

$\therefore x - \sqrt{5} = 0$ OR $x + \sqrt{5} = 0$

$\therefore x = \sqrt{5} = 0$ OR $x = -\sqrt{5}$

$\therefore$ Suppose $\alpha = \sqrt{5}$, $\beta = \sqrt{5}$

$a = 1$, $b = 0$, c $= -\sqrt{5}$

Sum of zeros $(\alpha + \beta) = (\sqrt{5}) + (-\sqrt{5}) = \sqrt{5} - \sqrt{5} = 0 = \dfrac{-0}{1} = \dfrac{-b}{a}$

Product of zeros $(\alpha \bullet \beta) = (\sqrt{5})(-\sqrt{5}) = -5 = \dfrac{-5}{1} = \dfrac{c}{a}$

39. Suppose $\alpha = 5 + \sqrt{3}$ and $\beta = 5 - \sqrt{3}$

$\alpha + \beta = 5 + \sqrt{3} + 5 \ \sqrt{3} = 10$

and $\alpha\ \beta = (5 + \sqrt{3})(5 - \sqrt{3}) = 25 - 3 = 22$

$\therefore$ The required quadratic polynomial

$= k\ [x^2 - (\alpha + \beta)\ x + \alpha \bullet \beta]$, K + 0, K $\in$ R

$= k\ (x^2 - 10\ x + 22)$

40. Here, the 7^{th} term exceeds the 5^{th} term by 12

$\therefore a_7 = a_5 + 12$

$\therefore a_7 - a_5 = 12$

$\therefore (a + 6d) - (a + 4d) = 12$

$\therefore a + 6d - a - 4d = 12$

$\therefore 2d = 12$

$\therefore d = 6$

Now, $a_3 = 16$

$\therefore\ a + 2d = 16$

$\therefore\ a + 2(6) = 16$

$\therefore\ a + 12 = 16$

$\therefore\ a = 16 - 12$

$\therefore\ a = 4$

$\therefore\ a_1 = a = 4$

$\therefore\ a_2 = a + d = 4 + 6 = 10$

$\therefore\ a_3 = a + 2d = 4 + 2(6) = 4 + 12 = 16$

Therefore, AP will be 4, 10, 16, 22, ...

41. Here, the number of logs in each row are in an AP. 20, 19, 18, ..., n terms, in which $S_n = 200$

$\therefore\ a = 20,\ d = 19 - 20 = -1,\ S_n = 200$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore\ 200 = \frac{n}{2}[2(20) + (n-1)(-1)]$$

$\therefore\ 400 = n(40 - n + 1)$

$\therefore\ 400 = n(41 - n)$

$\therefore\ 400 = 41n - n^2$

$\therefore\ n^2 - 41n + 400 = 0$

$\therefore\ n^2 - 25n - 16n + 400 = 0$

$\therefore\ n(n - 25) - 16(n - 25) = 0$

$\therefore\ (n - 25)(n - 16) = 0$

$\therefore\ n - 25 = 0$ OR $n - 16 = 0$

$\therefore\ n = 25$ OR $n = 16$

Now, $a_n = a + (n-1)d$

If $n = 25$,

$a_{25} = 20 + (25 - 1)(-1) = 20 - 24 = -4$

If $n = 16$,

$a_{16} = 20 + (16 - 1)(-1) = 20 - 15 = 5$

Here, the number of logs in the 16th row is 5 and the number of logs in the 25th row is – 4 negative, which is not possible

$\therefore$ Hence, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

42. Suppose, the point P $(x, 0)$ on the X-axis divides the line segment connecting the points A $(1, -5)$ and B $(-4, 5)$in the ratio $m_1 : m_2$.

The co-ordinates of the dividing point P = $\left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$$(x, 0) = \left(\frac{-4m_1 + m_2}{m_1 + m_2}, \frac{5m_1 - 5m_2}{m_1 + m_2}\right)$$

$$0 = \frac{5m_1 - 5m_2}{m_1 + m_2}$$ (comparing y co-ordinates)

$\therefore\ 0 = 5m_1 - 5m_2$

$\therefore\ 5m_1 = 5m_2$

$\therefore\ m_1 = m_2$

$\therefore\ \dfrac{m_1}{m_2} = \dfrac{1}{1}$

$\therefore\ m_1 : m_2 = 1 : 1$

$x = \dfrac{-4m_1 + m_2}{m_1 + m_2}$ (comparing x co-ordinates)

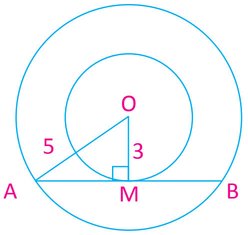
$\therefore\ x = \dfrac{-4(1)+1}{1+1}$

$\therefore\ x = \dfrac{-4+1}{2}$

$\therefore\ x = -\dfrac{3}{2}$

Thus, the x-axis divides the line segment connecting the points A (1, –5) and B (–4, 5) at point $\left(-\frac{3}{2}, 0\right)$ in a 1 : 1 ratio.

43.



Here, chord AB of ⊙ (0, 5) touches (0, 3) as point M.

Therefore, OM ⊥ AB and M is the midpoint of AB.

In Δ OMA; ∠OMA = 90°

$\therefore\ AM^2 + OM^2 = OA^2$ (Pythagoras Theorem)

$\therefore\ AM^2 + (3)^2 = (5)^2$

$\therefore\ AM^2 + 9 = 25$

$\therefore\ AM^2 = 25 - 9$

$\therefore\ AM^2 = 16$

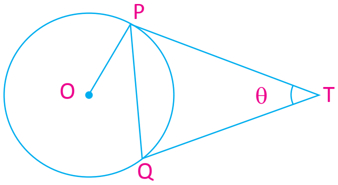
$\therefore\ AM = 4$

But, $AB = 2AM = 2 \times 4$

$\therefore\ AB = 8$

Hence, the length of chord AB is 8 cm.

44.



We are given a circle with centre O, an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.

Suppose, ∠PTQ = θ

Now, TP = TQ (theorem 10.2)

So, Δ TPQ is an isosceles triangle.

$\therefore \angle TPQ = \angle TQP = \frac{1}{2}(180° - \angle PTQ)$

$= \frac{1}{2}(180° - \theta)$

$= 90° - \frac{1}{2}\theta$

Now, $\angle OPT = 90°$ (theorem 10.1)

$\therefore \angle OPQ = \angle OPT - \angle TPQ$

$= 90° - \left(90° - \frac{1}{2}\theta\right)$

$= 90° - 90° + \frac{1}{2}\theta$

$= \frac{1}{2}\theta$

$\therefore \angle OPQ = \frac{1}{2}\angle PTQ$

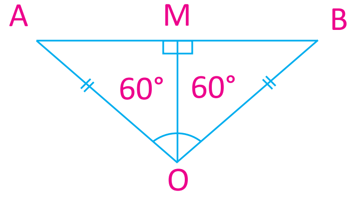
$\therefore \angle PTQ = 2\angle OPQ$

45. Area of the segment AYB $= \frac{\theta}{360} \times \pi r^2$

$= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21$

$= 462 \text{ cm}^2$

Now, draw OM $\perp$ AB.



In Δ AMO and Δ BMO,

$\angle$ AMO = $\angle$ BMO (Right Angle)

OA = OB

OM = OM (same side)

$\therefore \Delta AMO \cong \Delta BMO$ (RHS criterion)

So, M is the mid-point of AB and $\angle AOM = \angle BOM = \frac{1}{2}\angle AOB$

$= \frac{1}{2} \times 120° = 60°$

In Δ OMA,

$\cos 60° = \frac{OM}{OA}$ and $\sin 60° = \frac{AM}{OA}$

$\therefore \frac{1}{2} = \frac{OM}{21}$ $\therefore \frac{\sqrt{3}}{2} = \frac{AM}{21}$

$\therefore OM = \frac{21}{2}$ cm $\therefore AM = \frac{21\sqrt{3}}{2}$ cm

Now, AB = 2AM = $2 \times \frac{21\sqrt{3}}{2} = 21\sqrt{3}$ cm

Area of Δ OAB $= \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$

$= \frac{441}{4}\sqrt{3}$ cm²

Area of the segment AYB

$= 462 - \frac{441}{4}\sqrt{3}$

$= \frac{21}{4}(88 - 21\sqrt{3})$ cm²

46. Total number of outcomes = 5

(i) Suppose event A, when the queen is drawn and put aside.

$\therefore P(A) = \frac{\text{Number of queen card}}{\text{Total number of outcomes}}$

$\therefore P(A) = \frac{1}{5}$

(ii) When the queen is drawn and put aside the total number of remaining cards = 4

∴ Total number of outcomes = 4

(a) Suppose event B is getting of aces.

$\therefore P(B) = \frac{\text{Number of aces}}{\text{Total number of outcomes}}$

$\therefore P(B) = \frac{1}{4}$

(b) Suppose event C is getting queen.

$\therefore P(C) = \frac{\text{Number of queen}}{\text{Total number of outcomes}}$

$= \frac{0}{4}$

$\therefore P(C) = 0$

Section-D

47. Suppose, the unit digit is y and the tens digit of number is x

∴ Original number = $10x + y$

Now, when the digits are reversed y becomes the ten's digit and x become unit is digit.

∴ New number = $10y + x$

According to the first condition;

$x + y = 9$ ...(1)

According to the second condition;

$9(10x + y) = 2(10y + x)$

$\therefore 90x + 9y = 20y + 2x$

$\therefore 88x - 11y = 0$

$\therefore 8x - y = 0$ ...(2)

Add equation (1) & (2)

$x + y = 9$

$8x - y = 0$

$\therefore\ 9x = 9$

$\therefore\ x = 1$

Put $x = 1$ in equation (1)

$x + y = 9$

$\therefore\ 1 + y = 9$

$\therefore\ y = 8$

Original Number $= 10x + y$

$= 10\ (1) + 8$

$= 10 + 8$

$= 18$

Hence, the numbers is 18

48. Suppose, the speed of train $= x$ *km/h*

if the speed is less by 8 *km/h*, then the new speed = $(x - 8)$ *km/h*

Speed = $(x - 8)$ *km/h*

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time taken to travel 480 km at the original speed = $\frac{480}{x} \times h$

Time taken to travel 480 km at the new speed = $\frac{480}{x-8} \times h$

As per condition

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$\therefore\ 480\ x - 480\ (x - 8) = 3x\ (x - 8)$

$\therefore\ 480\ x - 480\ x + 3840 = 3x^2 - 24x$

$\therefore\ 0 = 3x^2 - 24x - 3840$

$\therefore\ x^2 - 8x - 1280 = a$

$\therefore\ x2 - 40x + 32x - 1280 = 0$

$\therefore\ x\ (x - 40) + 32\ (x - 40) = 0$

$\therefore\ x - 40 = 0$ OR $x + 32 = 0$

$\therefore\ x = 40$ OR $x = -32$

but x is original speed of train, so $x \neq -32$

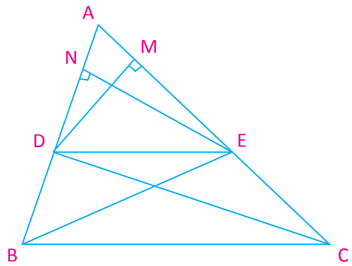
$x = 40$ *km/h*

The original speed of train is 40 *km/h*

49. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: In ABC, a line parallel to side BC intersects AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$



Proof : Join BE and CD and also draw DM $\perp$ AC and EN $\perp$ AB.

Then, $ADE = \frac{1}{2} \times AD \times EN,$

$BDE = \frac{1}{2} \times DB \times EN,$

$ADE = \frac{1}{2} \times AE \times DM$ and

$DEC = \frac{1}{2} \times EC \times DM.$

$$\therefore \frac{ADE}{BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad ...(1)$$

$$\text{and } \frac{ADE}{DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad ...(2)$$

Now, Δ BDE and Δ DEC are triangles on the same base DE and between the parallel BC and DE.

then, BDE = DEC ...(3)

Hence from eq^n. (1), (2) and (3),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

50. (i) $\Delta AMC \sim \Delta PNR$

$\Delta ABC \sim \Delta PQR$ (Given)

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \quad ...(1)$$

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ ...(2)

But CM and RN are medians,

AB = 2AM and PQ = 2PN

As per eq^n. (1), $\frac{AB}{PQ} = \frac{CA}{RP}$

$$\therefore \frac{2AM}{2PN} = \frac{CA}{RP}$$

$$\therefore \frac{AM}{PN} = \frac{CA}{RP} \quad ...(3)$$

As per eq^n. (2), $\angle A = \angle P$

$\therefore \angle MAC = \angle NPR$ ...(4)

As per eq^n. (3) and (4),

$\Delta AMC \sim \Delta PNR$ (SAS similarity) ...(5)

(ii) $\dfrac{CM}{RN} = \dfrac{AB}{PQ}$

As per eq^n. (5), $\dfrac{CM}{RN} = \dfrac{CA}{RP}$ ...(6)

But As per eq^n. (1), $\dfrac{CA}{RP} = \dfrac{AB}{PQ}$ ...(7)

$\therefore \dfrac{CM}{RN} = \dfrac{AB}{PQ}$ (From (6) and (7)) ...(8)

(iii) $\Delta CMB \sim \Delta RNQ$

As per eq^n. (1), $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$

From eq^n. (8), $\dfrac{CM}{RN} = \dfrac{BC}{QR}$ ...(9)

Now, $\dfrac{CM}{RN} = \dfrac{AB}{PQ}$ (From eq^n (8))

$\therefore \dfrac{CM}{RN} = \dfrac{2BM}{2QN}$

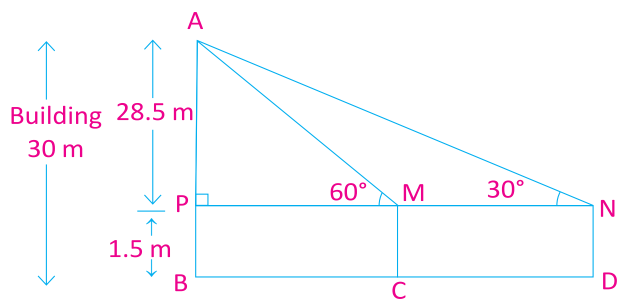
$\therefore \dfrac{CM}{RN} = \dfrac{BM}{QN}$ ...(10)

As per eq^n. (9) and (10),

$$\frac{CM}{RN} = \frac{BC}{QR} = \frac{BM}{QN}$$

$\therefore \Delta CMB \sim \Delta RNQ$ (SSS similarity)

51.



Here, AB is building, D is the starting point of the boy, C is the final place of the boy. N and M show the boy's eyes at this point, suppose the extended NM meets AB in P.

Therefore,

$\angle APM = \angle APN = 90°$, $\angle ANP = 30°$, $\angle AMP = 60°$,

$ND = MC = PB = 1.5$ and $AP = AB - PB = 30 - 1.5 = 28.5$ m

In Δ APM, $\angle APM = 90°$

$\therefore \tan 60° = \dfrac{AP}{PM}$

$\therefore \sqrt{3} = \dfrac{28.5}{PM}$

$\therefore PM = \dfrac{28.5}{\sqrt{3}}$ m

In Δ APN, $\angle APN = 90°$

$$\therefore \tan 30° = \frac{AP}{PN}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{28.5}{PN}$$

$$\therefore PN = 28.5\sqrt{3} \text{ m}$$

The boy at the distance = DC = NM = PN – PM

$$= 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}} = \frac{85.5 - 28.5}{\sqrt{3}}$$

$$= \frac{57}{\sqrt{3}} = 19\sqrt{3} \text{ m}$$

52. Side of cubical block = l = 7 cm

Diameter of hemisphere = 7 cm

$$\therefore r = \frac{7}{2} \text{ cm}$$

Total surface area of solid

= Surface area of cube + CSA of hemisphere – Area of base of hemisphere

$$= 6l^2 + 2\pi r^2 - \pi r^2$$

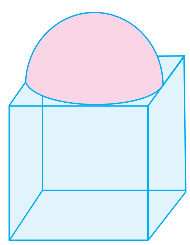
$$= 6l^2 + \pi r^2$$

$$= 6(7)^2 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 6(49) + \frac{77}{2}$$

$$= 294 + 38.5$$

$$= 332.5 \text{ cm}^2$$



53.

Cylinder	Cone
d = 3 cm.	d = 3 cm
$r = \frac{3}{2}$ cm	$\therefore r = \frac{3}{2}$ cm
H = (?)	h = 2 cm

Total length = 12 cm

Height of cylinder + 2 × Height of cone = 12

H + 2 × 2 = 12

$\therefore$ H + 4 = 12

$\therefore$ H = 12 – 4

$\therefore$ H = 8 cm

Volume of air = Volume of cylinder + 2 × Volume of cone

$$= \pi r^2 H + 2 \times \frac{1}{3}\pi r^2 h$$

$$= \pi r^2 \left(H + \frac{2h}{3}\right)$$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \left(8 + \frac{2 \times 2}{3}\right)$$

$$= \frac{198}{28} \times \left(\frac{24+4}{3}\right)$$

$$= \frac{198}{28} \times \frac{28}{3}$$

$$= 66 \text{ cm}^3$$

Hence, the volume of air contained in the model that Rachel made is 66 cm^3

54.

Class interval	Frequency
0 - 10	5
10 - 20	x
20 - 30	20
30 - 40	15
40 - 50	y
50 - 60	5
Total	60

class	frequency (f_i)	cf
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$

Here, M = 28.5

$n = 60$

Median class = 20 – 30

l = lower limit of median class = 20

n = total frequency = 60

cf = cumulative frequency of class preceding the median class = $5 + x$

f = frequency of median class = 20

h = class size = 10

$$M = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\therefore 28.5 = 20 + \left(\frac{\frac{60}{2} - (5 + x)}{20}\right) \times 10$$

$$\therefore 28.5 - 20 = \frac{(30 - 5 - x) \times 10}{20}$$

$$\therefore \frac{8.5 \times 20}{10} = 25 - x$$

$\therefore 17 = 25 - x$

$x = 25 - 17$

$x = 8$

Now, $\Sigma f_i = n = 60$

$\therefore 45 + x + y = 60$

$\therefore 45 + 8 + y = 60$

$\therefore 53 + y = 60$

$\therefore y = 60 - 53$

$\therefore y = 7$

Thus, $x = 8$ and $y = 7$.